

Tribhuvan University
Institute of Science and Technology

2069



Bachelor Level / First Year/ First Semester/ Science
Computer Science and Information Technology - MTH. 104
(Calculus and Analytical Geometry)

Full Marks: 80
Pass Marks: 32
Time: 3 hours.

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.
Attempt all questions.

Group A

(10x2=20)

1. Verify the mean value theorem for the function $f(x) = \sqrt{x(x-1)}$ in the interval $[0, 1]$. ✓
2. Find the length of the curve $y = \frac{4\sqrt{2}}{3}x^{3/2} - 1$ for $0 \leq x \leq 1$. ✓
3. Test the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n!}$ by comparison test. ✓
4. Obtain the semi-major axis, semi-minor axis, foci, vertices $\frac{x^2}{25} + \frac{y^2}{16} = 1$. ✓
5. Find the angle between the vectors $2\mathbf{i} + \mathbf{j} + \mathbf{k}$ and $-4\mathbf{i} + 3\mathbf{j} + \mathbf{k}$. ✓
6. Obtain the area of the region R bounded by $y = x$, and $y = x^2$ in the first quadrant. ✓
7. Show that the function

$$f(x, y) = \begin{cases} \frac{2xy}{x^2 + y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$$
is continuous at every point in the plane except the origin.
8. Using partial derivatives, find $\frac{dy}{dx}$ if $2xy + \tan y - 4y^2 = 0$. ✓
9. Verify that the partial differential equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = \frac{2z}{x}$ is satisfied by

$$z = \frac{1}{x} \phi(y-x) + \phi'(y-x).$$
10. Find the general solution of the equation ✓

$$x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = (x+y)z.$$

$$x^2 \cdot \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = (x+y)z$$

$$x(D^2 - D) = \frac{2z}{x}$$

$$x(D^2 - D) \frac{1}{x} = 2z$$

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Group B

(5x4=20)

- ✓ 11. State and prove mean value theorem for definite integral.
- ✓ 12. Find the area of the region that lies in the plane enclosed by the cardioid $r = 2(1 + \cos \theta)$.
- ✓ 13. What do you mean by principal unit normal vector? Find unit tangent vector and principal unit normal vector for the circular motion $\vec{r}(t) = (\cos 2t)\mathbf{i} + (\sin 2t)\mathbf{j}$.
- ✓ 14. Define partial derivative of a function $f(x, y)$ with respect to x at the point (x_0, y_0) . State Euler's theorem, verify it for the function

$$F(x, y) = x^2 + 5xy + \sin x + 7e^x \quad x = \left(\frac{y}{2}\right) + 1.$$

- ✓ 15. Find a particular integral of the equation

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial z}{\partial y} = 2y - x^2.$$

Group C

(5x8=40)

- ✓ 16. Graph the function $y = x^{2/3} - 5x^{1/3}$.
- ✓ 17. What is meant by Maclaurin series? Obtain the Maclaurin series for the function $f(x) = e^{-x}$.

- ✓ 18. Evaluate the double integral $\int_0^1 \int_{y/2}^{y/2+1} \frac{2x-y}{2} dx dy$ by applying the transformation

$$u = \frac{2x-y}{2}, v = \frac{y}{2} \text{ and integrating over an appropriate region in the } uv\text{-plane.}$$

- ✓ 19. Define maximum and minimum of a function at a point. Find the local maximum and local minimum of the function $f(x, y) = 2xy - 5x^2 - 2y^2 + 4x + 4y - 4$.

OR

Find the volume of the region D enclosed by the surface $z = x^2 + y^2$ and $z = 8 - x^2 - y^2$.

- ✓ 20. Find the solution of the equation

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y$$

OR

Find the particular integral of the equation $(D^2 - D')z = 2y - x^2$

$$\text{Where } D = \frac{\partial}{\partial x}, \quad D' = \frac{\partial}{\partial y}.$$